Wondering What Might Be

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Abstract

This paper explores the possibility of supplementing the suppositional view of indicative conditionals with a corresponding view of epistemic modals. The most striking feature of the suppositional view consists in its claim that indicative conditionals are to be evaluated by conditional probabilities. On the basis of a natural link between indicative conditionals and epistemic modals, a corresponding thesis about the probabilities of statements governed by epistemic modals can be derived. The paper proceeds by deriving further consequences of this thesis, in particular, the logic of epistemic modals and their logical interaction with indicative conditionals are studied.

1 Introduction

The recent debate about epistemic modals focuses mainly on two rival positions: contextualism and relativism. Within the debate about indicative conditionals, there exists a promising alternative strategy: the suppositional view of conditionals. On this view, indicative conditionals do not express categorical statements. Rather, they are used to make conditional assertions and are evaluated by conditional probabilities. Can a corresponding view be developed for epistemic modals?

Prima facie, there is evidence that indicative conditionals and epistemic modals should allow for a unified treatment. There seem to be strong logical connections between the two kinds of expressions which, as will be explained below, imply that epistemic modality allows for an equivalent expression in terms of indicative conditionals. She *might* be at the party. *If* she is at the party, she will be seeing Jack. Hence, she *might* be seeing Jack. Consider also the following inference. She *must* be either in New York or in L.A. So, *if* she is not in New York, she is in L.A. On the assumption that indicative conditionals are evaluated by conditional probabilities, the following question arises: how are statements governed by epistemic modals to be evaluated in order for these inferences to be reasonable?

I will proceed as follows. Starting with a brief outline of the suppositional view and presenting the core inferences connecting epistemic modals with indicative conditionals, I will develop a hypothesis about the subjective probabilities of epistemic 'might'-statements. Thereafter, the subjective nature of this constraint will be discussed. Finally, I will focus on a probabilistic notion of validity on the basis of which the logic of epistemic modals and their logical interaction with indicative conditionals can be described.

2 The Suppositional View

On the suppositional view, the statement of an indicative conditional is a conditional statement: the consequent is stated within the scope of the supposition of the antecedent. A sincere utterance of a conditional is not described as a categorical assertion of a proposition but rather as a conditional assertion of the consequent under the supposition of the antecedent (a similar story is told for many other speech acts). Analogously, epistemic attitudes towards conditionals are reinterpreted as certain kinds of conditional attitudes. At its center, the suppositional view claims that conditionals are to be evaluated by conditional probabilities: the probability of a conditional is the conditional probability of the consequent given the antecedent. According to this picture, we value a conditional highly just in case the conditional probability of the consequent given the antecedent is high.[1]

Let us make the main thesis of the suppositional view a bit more precise. I will represent the indicative conditional by the double arrow '$\Rightarrow$'. Moreover, let us call a sentence *non-conditional* if it does not contain the double arrow. A conditional is said to be *simple* if both its antecedent and its consequent are non-conditional. The main thesis of the suppositional view applies to simple conditionals only. Following Edgington (1995), I will call it *The Thesis*:

> **(The Thesis)**
> Let P be a rational credence function and A and B non-conditional sentences. Then $P(A \Rightarrow B)$ equals the conditional probability $P(B|A)$.

Usually, the conditional probability $P(B|A)$ is defined as $\frac{P(A \wedge B)}{P(A)}$ for $P(A) > 0$, and is left undefined for $P(A) = 0$. Nonetheless, within the debate about conditionals it is an attractive option to set $P(B|A) = 1$ if $P(A) = 0$.[2] Given The Thesis, this would correspond to the idea that an indicative conditional is vacuously acceptable if its antecedent is assigned credence 0. Compare the view that a counterfactual is vacuously true if its antecedent is necessarily false. For the purpose of this paper, I will assume that this way of evaluating conditionals with epistemically impossible antecedents is correct.

3 A Logical Link

One may have the impression that indicative conditionals are epistemic or subjective in a way similar to epistemic modals. This impression can be substantiated by drawing attention to inferences connecting epistemic modals and in-

[1] For a defence of the suppositional view, see e.g. Adams (1975), Barnett (2006), and Edgington (1995).

[2] Adams (1996: 2f.) takes it to be the default assumption.

dicative conditionals. I will focus on two argument patterns which jointly imply certain equivalences between epistemic modals and indicative conditionals.

To begin with, consider the following pattern of inference:

(1) It might be that they lost last night's competition.

(2) If they lost last night's competition, they fired the coach.

(3) Therefore, it might be that they fired the coach.

In the light of such inferences, one may conjecture that indicative conditionals *preserve* epistemic possibility. If we represent epistemic possibility by the diamond '$\Diamond$', this inference can be formalized as

$$\Diamond A, A \Rightarrow B \therefore \Diamond B \,.$$

Let us call this pattern of inference PRESERVATION.[3]

A second pattern of inference deserves our attention:

(4) Certainly, either the gardener or the butler did it.

(5) Therefore, if the gardener didn't do it, the butler did.

In this inference, we infer the indicative conditional from the corresponding strict epistemic conditional (here the disjunction in (4) is described as a material conditional). This inference can be formalized like this (the horseshoe, '$\supset$', represents the material conditional, the box, '$\Box$', is to be interpreted as epistemic necessity):

$$\Box(A \supset B) \therefore A \Rightarrow B \,.$$

We will refer to this pattern as STRICTNESS. If this pattern of inference is valid (as it appears to be), indicative conditionals cannot be stronger than the corresponding strict epistemic conditionals.[4]

On the basis of such *prima facie* valid inferences, it seems worthwhile to investigate the conjecture that there is an intimate link between epistemic modals and indicative conditionals. Assuming epistemic modals to satisfy the modal logic K, the weakest normal modal logic, one can show on the basis of PRESERVATION and STRICTNESS the following two equivalences to hold ('$\equiv$' represents material equivalence):

$$\Diamond A \equiv \neg(A \Rightarrow \bot), \text{ and } \Box A \equiv \neg A \Rightarrow \bot \;.$$

As is easily seen, these two equivalences turn the box and the diamond into two dual modal operators. Similar equivalences have first been introduced by

[3]See Adams (1996: 12f.) for a brief discussion of how this pattern may relate to a certain preservation property of *modus ponens* (the preservation of positive probability); Bradley (2000) contains a more extensive treatment of this latter property and shows how it gives rise to a new 'triviality result' for the conditional.

[4]A variant of STRICTNESS is defended in Edgington (1986), where it is pointed out that even though the indicative conditional is not implied by the material conditional as such, it is implied by a material conditional which is taken to be certain.

Stalnaker and Lewis.[5] Recently, they have attracted a great deal of attention in modal epistemology (where counterfactuals are related to metaphysical modality).[6] Since the proofs of the two equivalences are similar, let us only review one of them, the equivalence of $\Box A$ and $\neg A \Rightarrow \bot$, say. So, assume $\Box A$. In K, this implies $\Box(\neg A \supset \bot)$. By STRICTNESS, we get $\neg A \Rightarrow \bot$. Conversely, assume the latter. For reductio, suppose $\neg\Box A$, i.e. $\Diamond\neg A$. By PRESERVATION, we can infer $\Diamond \bot$ from these two assumption. But $\neg\Diamond \bot$ is a theorem in K, so we can negate one of our assumptions. Hence, $\Box A$ follows.

If these two equivalences are correct, epistemic modality allows for an equivalent expression in terms of indicative conditionals.[7] Something would be epistemically possible if assuming it to be true does not lead one to a contradiction. Correspondingly, something would be epistemically necessary if assuming it to be false leads one to a contradiction.

4 Might and Must

The idea is now to apply The Thesis to the equivalences between indicative conditionals and statements involving epistemic modals, i.e. the idea is that $\Diamond A$ should have the same subjective probability as $\neg(A \Rightarrow \bot)$ and $\Box A$ should be evaluated in the same way as $\neg A \Rightarrow \bot$, since they are logically equivalent, and statements which are logically equivalent should be assigned the same subjective probability.

Adopting this strategy, one arrives at the following two principles:

> (MIGHT)
> Let P be a rational credence function and A a (non-conditional) sentence. Then $P(\Diamond A) = 1$ just in case $P(A) > 0$ and $P(\Diamond A) = 0$ just in case $P(A) = 0$.
>
> (MUST)
> Let P be a rational credence function and A a (non-conditional) sentence. Then $P(\Box A) = 1$ just in case $P(A) = 1$ and $P(\Box A) = 0$ just in case $P(A) < 1$.

There is one minor complication. The Thesis is only concerned with simple conditionals. As it stands, it is not applicable to negations of conditionals such as $\neg(A \Rightarrow \bot)$. One possibility would be to first apply The Thesis to $\neg A \Rightarrow \bot$ in order to evaluate $\Box A$ and then exploit the duality of the modal operators to evaluate $\Diamond A$. Alternatively, a straightforward extension of The Thesis can be used which covers negations of conditionals as well. The standard laws of probability require that $P(\neg A) = 1 - P(A)$. Applied to conditionals, this means that $P(\neg(A \Rightarrow B)) = 1 - P(A \Rightarrow B)$. The same result would follow.

Now to the derivations. Since they are similar, I will only present the derivation of MIGHT, and I will use the second strategy mentioned above. So,

[5] Cf. Lewis (1973: 22) and Stalnaker (1968).

[6] See Williamson (2007: ch. 5).

[7] This does not imply that epistemic modals are synonymous with the corresponding conditional constructions; they may well be primitive expressions.

suppose $P(\Diamond A) = 1$. Given the corresponding equivalence, this is equivalent to $P(\neg(A \Rightarrow \bot)) = 1$. By the laws of probability, this is equivalent to $P(A \Rightarrow \bot) = 0$. Given The Thesis, this means that $P(\bot\,|A) = 0$. Using the definition of conditional probability as suggested above, this is seen to be equivalent to $P(A) > 0$ as desired. Now suppose $P(\Diamond A) = 0$. By a similar argument, this can be shown to be equivalent to $P(\bot\,|A) = 1$. Again, using the suggested definition of conditional probability, this is equivalent to $P(A) = 0$, and we are done.[8]

What is stated by MIGHT and MUST? By relating the credence in a sentence governed by an epistemic modal to the credence in the embedded sentence, they impose a certain coherency constraint. MIGHT requires that one should accept 'It might be that *p*' just in case one's credence in '*p*' is positive and that one should reject it if one's credence is 0.[9] Similarly, MUST demands to accept 'It must be that *p*' (or 'Certainly, *p*') just in case one is certain that *p* and to reject it otherwise.

One observation which tells in favour of MIGHT is that 'might'-statements do not seem to come in degrees. Acceptance of them appears to be an all-or-nothing matter.[10] For instance, if 'might'-statements were to come in degrees, 'probably' should be attachable to a 'might'-statement in an interesting way. But the following two sentences seem to be odd:

(6) Probably, it might be that they are at home.

(7) It is more likely that it might be that they are at home than that it might be that they are away.

In what follows, I will restrict myself to a discussion of MIGHT, but similar points will mostly be taken to apply to MUST as well.[11] In English, various notions may be taken to have a use in which they express epistemic possibility. For instance, 'maybe', 'perhaps', 'might be', 'possibly', and 'can/could be' all seem to allow for a use as an epistemic modal. To simplify the issue, I will focus on epistemic uses of 'might', but I am inclined to think that it belongs to a larger family of modal expressions which can all be used to express the same kind of epistemic modality.

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[8] A word on the restriction of MIGHT and MUST to non-conditional sentences. I have bracketed this condition, since it does not seem to be necessary. It was needed in order to derive the two constraints from The Thesis which is restricted in this way.

[9] If the space of possibilities is infinite, the association of possibility with positive probability cannot be implemented in standard probability theory. Hence, I anticipate that one will ultimately need to use some kind of non-standard probabilities. But as a first approximation, standard probabilities seem to be good enough.

[10] Let us add, though, that in a given conversation it may depend on context what we treat as having positive probability.

[11] A note of caution. It seems that in asserting a 'must'-sentence we indicate that the corresponding belief is inferred rather than gained in a direct way. This feature of 'must' is not captured by MUST; for present purpose, I assume that it allows for a pragmatic explanation.

5 Subjectivity

One of the central issues in the debate about epistemic modals concerns the problem of *relevance*. Whose epistemic states are relevant for the evaluation of 'might'-sentences? The ultimate shape of this problem varies with the basic approach one is following. Within a contextualist framework, one needs to answer the question of whose epistemic states constitute the contextual factors according to which the truth conditions of 'might'-utterances are defined.[12] Due to the problem of relevance, theories of epistemic modals have become exceedingly complex. Contextualists have been forced into postulating increasingly high degrees of flexibility in the ways a 'might'-utterance may depend on context. More radical contextualists have even gone so far to give up the very idea that a unique proposition is expressed by a given 'might'-utterance.[13] The relativist faces the analogous problem of determining the epistemic states which constitute the point of evaluation relative to which a given 'might'-utterance is said to be true or false.[14]

Might implies that we should accept a 'might'-statement just in case we have positive credence in the embedded sentence. If this is so, it has direct consequences for the problem of relevance: only one's own epistemic state is relevant for the evaluation of epistemic 'might'-statements.

At first glance, the solipsistic aspect of Might accords quite well with linguistic usage. Take a case where one should be quite uncertain about what is known or believed by other people. For example, suppose that I have no conclusive evidence as to whether N.N. can rule out 'p' or not. So, for all I know, N.N. might know that 'p' is false. Yet I have positive credence in 'p'. It seems that I should be ready to accept (and to assert) that it might be that p. Now, were N.N.'s epistemic state relevant for the evaluation of this statement, one would expect me to be prepared to say something like 'I do not know whether it might be that p because I am uncertain about what N.N. may know'. However, this kind of linguistic behaviour is not what we display in situations like these. If I have some evidence for the proposition that p, I will simply accept that it might be that p. My acceptance is not undermined by my uncertainty about the epistemic state of some other (contextually relevant) epistemic subject.

Such a view diminishes the possibility of taking an agnostic stance towards a 'might'-statement. Hence, one needs an explanation of what is going on in cases which seem to show that an agnostic stance towards 'might'-statements is sometimes mandatory. I will suggest that in such situations, 'might' is used to express a more objective kind of possibility. I will first give an example which seems to be best explained by assuming 'might' to express *present objective chance*. Thereafter, I will consider cases which render the assumption plausible that 'might' can also be used to express *statistical chance*. Usually, the latter cases have been taken to fall in the range of an epistemic use of 'might', since

[12] Contextual accounts are defended in DeRose (1991), Hacking (1967), and Teller (1972).

[13] See von Fintel & Gillies (2007).

[14] Relativist positions are developed in Egan & Hawthorne & Weatherson (2005), Egan (2007), and MacFarlane (2006). A critique of relativism can be found in von Fintel & Gillies (2008).

they cannot be explained in terms of objective chances. The possibility of interpreting them in terms of statistical chances has not been considered. I will conclude with some more general considerations concerning the methodological integrity of postulating an ambiguity of 'might'.

There is an old chance device by which the lucky number of a lottery is determined. Due to its age, the device sometimes fails to register a certain number as input, i.e. it sometimes determines the lucky number from a limited range of numbers. I consider the number of my lottery ticket. As a result of my ignorance, I accept that

(8) My ticket might win.

Now, someone may object by reminding me of the occasional defect of the chance device. On the basis of the fact that there may be no present objective chance that my ticket will win, I might well say that

(9) I do not know whether my ticket might win.

The two sentences are not jointly assertable, for their conjunction would be Moore-paradoxical. Hence, it seems that 'might' expresses different conceptions in these two utterances. Nothing should prevent us from interpreting the first occurrence of 'might' epistemically, since it was made simply on the basis of my ignorance. The second utterance is made, however, on the grounds that I cannot exclude the possibility that my number does not have any objective chance of winning. The grounds I offer for my hesitation strongly suggest that I take 'might' in this case to express present objective chance or possibility. This explanation is much more straightforward than the attempt to extend the epistemic use in a way which is flexible enough to include the relevant facts about the chance device.

Now to one of DeRose's (1991) examples. A cancer test will be run. If its result is positive, there is a statistical chance that N.N. has cancer. If it is not, there is no such chance. Since I am ignorant about the test results, I accept that

(10) It might be that N.N. has cancer.

However, in a way similar to the case above, I may be forced to admit that

(11) I do not know whether N.N. might have cancer,

on the grounds that the test results may be negative. Now, a negative test result would mean that there is no statistical chance for N.N. having cancer. So, the reasons I have for taking an agnostic stance towards the 'might'-statement suggest that I take 'might' to express some kind of statistical possibility.[15]

In general, postulating ambiguity weakens a theory, for it makes — other things being equal — the theory more complex. However, attempts to provide a unified theory of 'might' have not gained in simplicity by sticking to the

[15]The conception of statistical chances alluded to here seems to be similar to the conception of evidential probability developed in Williamson (2000).

thesis that 'might' is unambiguous. There seems to be a trade-off between the ambiguity thesis and the complexity of a theory of epistemic modals. After all, we may gain in overall simplicity by assuming 'might' to be ambiguous. Taking 'might' to be ambiguous does not mean that the different possible meanings do not share a common pattern. On the present view, a similar structure underlays all uses of 'might': they all go by a certain kind of chances (subjective, objective, statistical).

6 A Logic for Epistemic Modals

As it stands, MIGHT is silent with respect to the truth conditions of statements governed by epistemic modals, or whether they have any. Hence, it does not provide an answer to the question of which arguments involving epistemic modals are truth preserving. The suppositional view of indicative conditionals faces a similar problem. Given that it does not specify truth conditions for indicative conditionals (and is perhaps even incompatible with them having truth conditions), it does not determine a logic for indicative conditionals which is based on the classical notion of logical consequence defined in terms of truth preservation. However, proponents of the suppositional view have used a probabilistic notion of validity, which was developed by Adams, to characterize the logic of indicative conditionals.[16] In a sense to be made precise, an argument is said to be valid just in case it preserves subjective probability. In this section, I will show how the probabilistic notion of validity can be extended on the basis of MIGHT to cover statements containing epistemic modals. The logic of epistemic modals then turns out to be a surprisingly strong one: S5.

To begin with, let us review the motivating idea behind Adams's probabilistic notion of validity. It can best be put in terms of *uncertainty*: given a credence function P, there is a corresponding uncertainty function u_P defined as $1 - P$, i.e. the uncertainty $u_P(A)$ of a proposition A equals $1 - P(A) = P(\neg A)$. So, the less likely a proposition, the more uncertain it is. Given the standard laws of probability (which will be reviewed shortly), Adams observed that an argument (with finitely many premises) is classically valid just in case the uncertainty of the conclusion never exceeds the sum of the uncertainties of the premises.[17] This is perhaps best illustrated by considering the special case of arguments with a single premise. With respect to such arguments, Adams's observation implies that they are classically valid just in case the premise is always at least as uncertain as the conclusion, or, to put it another way, the premise's subjective probability is never greater than the probability of the conclusion. Informally, Adams observation corresponds to the claim that it is not rational to assign to a statement a higher subjective probability than to one of its logical consequences (if one is aware of the fact that they are logical consequences).

How do sums of uncertainty enter the stage? The basic observation here is this: the subjective probability can go down when one moves from two statements to their conjunction. I may be 0.99 certain that ticket 1 will lose, and I

[16] See Adams (1975) and Adams (1998).
[17] Cf. Adams (1998: ch. 7).

may be 0.99 certain that ticket 2 will lose. In the most natural circumstances, I should then be 0.98 certain that both tickets lose. Note that this value corresponds precisely to the sum of the uncertainties of the two premises. As one sees on reflection, a conjunction can never be more uncertain than the sum of the uncertainties of its conjuncts. Add to this the fact that an argument from finitely many premises to a conclusion is classically valid just in case the argument from the conjunction of the premises to the same conclusion is valid. Taken together, this motivates Adams's observation that the uncertainty of the conclusion of a valid argument cannot exceed the sum of the uncertainties of the premises.

Now to Adams's strategy of describing the logic of indicative conditionals in probabilistic terms. Once we have a characterization of classical validity in terms of subjective probability, we can try to extend this conception to cover indicative conditionals as well. Just add to the standard probabilistic laws the requirement that The Thesis must be satisfied. This yields an extended notion of a rational credence function for a language which contains indicative conditionals. Define a corresponding probabilistic notion of validity by saying that an argument (which may contain indicative conditionals) is probabilistically valid just in case the uncertainty of the conclusion never exceeds the sum of the uncertainties of the premises. Then one can study the logic of indicative conditionals in terms of their subjective probabilities. In what follows, I will apply this strategy to epistemic modals by using Might instead of The Thesis.

Consider a propositional language made up in the usual way from sentence letters $p_1, p_2, \ldots$, a complete set of truth-functional connectives, and the diamond '$\Diamond$' (think of it as representing 'It might be that'). Call this a *modal language*. Let us say that a function P over a modal language is a *standard probability function* if it satisfies the standard laws of probability, i.e. if

(i) $0 \leq P(A) \leq 1$ for all sentences A.

(ii) $P(T) = 1$ for every truth-functional tautology T.

(iii) $P(A \vee B) = P(A) + P(B)$ if $\neg(A \wedge B)$ is a truth-functional tautology.

Moreover, a standard probability function P over a modal language *satisfies* Might just in case the following condition holds:

(iv) $P(\Diamond A) = 1$ iff $P(A) > 0$ and $P(\Diamond A) = 0$ iff $P(A) = 0$.

Based on this, an extended notion of a credence function can be defined. Let us say that P is an *extended credence function over a modal language* if it is a standard probability function and satisfies Might. Now, the core notion of a probabilistically valid argument within a modal language can be defined:

> **(Df. Probabilistic Validity)**
> An argument $\phi_1, \ldots, \phi_n \therefore \psi$ within a modal language is said to be *probabilistically valid* iff there is no extended credence function P over this language such that $u_P(\psi) > \sum_{i=1}^{n} u_P(\phi_i)$.

So, an argument involving epistemic modals is said to be probabilistically valid iff the uncertainty of the conclusion cannot exceed the uncertainties of the premises, where the relevant credence functions is required to satisfy Might together with the standard laws of probability.

As a special case, a probabilistic notion of theoremhood can be derived. One can think of theorems as conclusions of valid arguments without any premises. Using the convention that empty sums equal zero, an argument without any premises is probabilistically valid just in case every extended credence function assigns subjective probability 1 to its conclusion. Based on this observation, let us call a sentence of a modal language *always acceptable* iff every extended credence function over this language assigns 1 to it. The notion of an always acceptable sentence can be seen to be a reasonable notion of a theorem on independent grounds, for a sentence of a truth-functional language is a theorem within classical logic just in case any standard probability function assigns 1 to it. Since every extended credence function is, by definition, a standard probability function, the notion of an always acceptable sentence is an extension of the classical conception of a theorem. Also, the notion of an always acceptable sentence is precisely the one which is needed to prove the following deduction theorem: an argument $\phi_1, \ldots, \phi_n \therefore \psi$ over a modal language is probabilistically valid just in case the material conditional $(\phi_1 \wedge \ldots \wedge \phi_n) \supset \psi$ is always acceptable (a proof of this fact is given in the appendix). For these reasons, the notion of an always acceptable sentence supplements probabilistic validity with an adequate conception of theoremhood.

Interestingly, one finds the following theorem to hold:

> **(Soundness and Completeness)**
> A sentence A of a modal language is always acceptable just in case A is a theorem of S5.

The proof of this theorem can be found in the appendix. But at least the soundness part of it can be motivated quite straightforwardly. By Might, attaching a modal operator to a sentence yields either 1 or 0; and attaching a modal operator to a sentence with values 1 or 0 yields the same values again, i.e. 1 if it was 1 before and 0 otherwise. From this one can infer that iterating modal operators does not make a difference, i.e. a formula always gets the value of the embedded formula which starts with the innermost modal operator. This indicates why every theorem of S5 is always acceptable.

To get a feeling of how the notion of an always acceptable sentence is to be applied, here are two examples.

(12) It might be that they are at home or it might be that they are not at home.

(13) It might be that they are at home and it might be that they are not at home.

Sentence (12) is an always acceptable sentence because you will either have positive credence in the thought that they are at home, or you will have positive credence in the thought that they are not at home. Hence, you will assign credence 1 to one of the disjuncts and thus to the disjunction as well. Sentence (13), on the other hand, is not an always acceptable sentence. For, you may be certain that they are at home. In such a situation, you should reject that they might be not at home. Consequently, you should reject the whole conjunction.

A strong logic for epistemic modals seems to be supported by the linguistic data. We do not find interesting iterations of epistemic modals within natural language. For instance, consider the following sentences:

(14) It might be that it might be that they are at home.

(15) It might be that it must be that they are at home.

Given that the logic of epistemic modals is as strong as S5, one can explain why these iterations hardly ever occur: they do not extend the expressive power which is already available without them. However, a strong logic for epistemic modals may have theoretical costs. But before we turn to a discussion of this issue, let us see how the logical interaction between indicative conditionals and epistemic modals can be studied on the basis of The Thesis and Might.

7 The Interaction with Indicative Conditionals

As remarked earlier, there seem to exist logical links between epistemic modals and indicative conditionals which suggest that these locutions should be studied in a unified framework. In particular, given that we can describe both the logic of epistemic modals and the logic of indicative conditionals in probabilistic terms, one would wish to unify these two branches of probabilistic logic in order to arrive at an account which can also describe the logical interactions between them. This section shows how this can be done. The basic idea will be simple: one only needs to extend the notion of probabilistic validity in a way which makes it applicable to a language containing both indicative conditionals and epistemic modals. This can be achieved by adding The Thesis together with Might to the standard laws of probability.

Let us call a modal language enriched by all simple conditionals which do not contain epistemic modals a *modal and conditional language*. Such a language contains neither conjunctions nor negations of conditionals; by definition, it also contains neither nested conditionals nor conditionals containing epistemic modals. I consider this restricted language in order to set aside all issues concerning the problem of embedded conditionals.

Let us say that a function P is an *extended credence function over a modal and conditional language* if it is an extended credence function over the non-conditional sentences, i.e. if it satisfies clauses (i) - (iv) above, and satisfies The Thesis, i.e. if it satisfies in addition the following condition:

(v) $P(A \Rightarrow B) = P(B|A)$, where $P(B|A) = P(A \wedge B)/P(A)$ if $P(A) > 0$ and $P(B|A) = 1$ otherwise.

Now, the notion of probabilistic validity carries over to a modal and conditional language: a (finite) argument is *probabilistically valid* iff there is no extended credence function over this language such that the uncertainty of the conclusion exceeds the sum of the uncertainties of the premises.

On the basis of a unified notion of validity within a probabilistic framework, one can now study the logical interaction between epistemic modals and indicative conditionals. Since MIGHT was derived from The Thesis by exploiting a joint consequence of PRESERVATION and STRICTNESS, it comes as no surprise that these inference patterns turn out to be probabilistically valid. To get an impression of how probabilistic validity connects the logic of epistemic modals with the logic of indicative conditionals, let us run through PRESERVATION and STRICTNESS with our initial examples. Recall that PRESERVATION is the inference pattern $\Diamond A, A \Rightarrow B \therefore \Diamond B$ and STRICTNESS is $\Box(A \supset B) \therefore A \Rightarrow B$. Start with PRESERVATION:

(16) It might be that they lost last night's competition.

(17) If they lost last night's competition, they fired the coach.

(18) Therefore, it might be that they fired the coach.

In order to show that this inference is probabilistically valid, we need to show that the uncertainty of the conclusion cannot exceed the sum of the uncertainties of the premises. By MIGHT, the probability of the conclusion is either 1 or 0. If it is 1, the corresponding uncertainty is 0, and we are done because 0 cannot exceed the sum of any non-negative numbers. If it is 0, the uncertainty of the conclusion is 1. By MIGHT, we infer that the probability of 'They fired the coach is zero'. Application of The Thesis to (17) yields that its probability is either 0 or 1. In the former case, its uncertainty is 1 and we are done, since the sum of the uncertainty of the premises will at least be 1. In the latter case, it follows that the probability of 'They lost last night's competition' is 0. By MIGHT, we can infer that the probability of (16) is 0, i.e. its uncertainty is 1, and we are done for the same reason as above.

Now to STRICTNESS. Recall the following inference:

(19) Certainly, either the gardener or the butler did it.

(20) Therefore, if the gardener didn't do it, the butler did.

Suppose we have a high credence in the premise. By MUST, this requires to assign probability 1 to the proposition that either the gardener or the butler did it. But this implies that the conditional probability that the butler did it given that the gardener didn't do it must be 1, too, by the standard laws of probability. Hence, this inference is probabilistically valid.

Thus, one can describe the joint logic of epistemic modals and indicative conditionals in a probabilistic framework. Adding MIGHT and The Thesis to the standard laws of probability allows one to formulate a probabilistic notion of validity which is broad enough to cover both epistemic modals and indicative conditionals. As expected, this logic validates PRESERVATION and STRICTNESS.

8 Concluding Remarks

A strong logic for epistemic modals such as S5 is at odds with popular accounts of epistemic modality. On standard approaches, 'might'-sentences are construed as being about one or many epistemic states. For instance, on a contextualist approach to epistemic modality, it is usually held that my utterance of 'It might be that they won last night' expresses a proposition which implies that I do not know that they lost last night. Accordingly, iterated 'might'-statements will be taken to involve iterated ascriptions of epistemic attitudes. So, one would take my utterance of 'It must be that it might be that they won last night' to express a proposition which implies that I know that I do not know that they lost last night. Yet the logic of epistemic attitudes such as knowledge or belief is probably much weaker than S5.[18] Hence, MIGHT seems to conflict with the view that 'might'-statements are about epistemic states, for MIGHT describes 'it might be that' as an operator governed by a logic stronger than the logic of the candidate epistemic attitudes.

So, by the lights of MIGHT, 'might'-statements do not involve the ascription of an epistemic attitude. In particular, MIGHT does not construe my utterance of a 'might'-sentence as expressing the proposition that I have positive credence in the embedded sentence. Rather, MIGHT describes the acceptability conditions of 'might'-sentences as *going by* positive credence in the embedded sentence: one should accept that it might be that p just in case one has positive credence in the proposition that p. This differs from the requirement to accept a 'might'-sentence just in case one is (fairly) certain that one has positive credence in the relevant proposition.

This feature of MIGHT stems from a corresponding property of The Thesis according to which the credence in a conditional should go by the conditional probability of the consequent given the antecedent. On its most natural interpretation, The Thesis does not construe conditionals as being about our epistemic states. It imposes a certain constraint on how our credences in conditionals should cohere with our credences in non-conditional propositions. If claims of epistemic modality are expressible in terms of indicative conditionals, it is much less surprising that 'might'-statements may go by positive credence without being about certain epistemic states in the same way as indicative conditionals may go by conditional probabilities without being about them.

The idea that indicative conditionals are not about our epistemic states is *prima facie* quite plausible. Now, if there are direct logical links between indicative conditionals and epistemic modals as suggested by PRESERVATION and STRICTNESS, we should either expect both indicative conditionals and epistemic modals to be about epistemic states or neither of them. Consequently, if one is inclined to construe indicative conditionals as not being about epistemic states, as suggested by The Thesis, one has reason to think that 'might'-statements are not about epistemic states either.

If 'might'-statement are not about epistemic states, it is an open question whether they can be assigned truth conditions compatible with MIGHT. At least

[18]See Williamson (2000: ch. 5) for detailed arguments.

to some extent, the problem of finding truth conditions compatible with Might is inherited from The Thesis. As various forms of the triviality results suggest, it is very hard to assign conditionals truth conditions which satisfy The Thesis in a systematic way.[19] If it is possible at all, indicatives will be highly sensitive to one's own epistemic state.[20] Of course, if one has a way of reconciling The Thesis with a certain account of truth conditions, one will be able to use it to derive a corresponding account of truth conditions for 'might'-sentences which satisfies Might by drawing on the equivalences between epistemic modals and indicative conditionals.[21]

On the other hand, many proponents of The Thesis have taken it to be best explained on a suppositional view of indicative conditionals.[22] According to such a view, the sincere utterance of an indicative conditional is a conditional assertion: the consequent is asserted within the scope of the supposition of the antecedent. Similarly, believing a conditional is a conditional belief: the consequent is believed on the assumption that the antecedent is true. It seems worthwhile to consider the thought that something similar may be true of epistemic modals. An assertion of a 'might'-statement would then be construed not as a categorical assertion of a truth conditional proposition but rather as a partial assertion of the embedded proposition.[23] Correspondingly, believing a 'might'-statement would not be construed as a higher-order belief about one's own (and perhaps others) epistemic state(s), but rather as a partial belief in the embedded sentence, i.e. as having positive credence in the embedded sentence. Thus, epistemic uses of 'it might be that' would be seen as modifying the speech act and the kind of epistemic attitude we are taking towards the embedded sentence in a way similar to how 'if' is described within the suppositional picture.

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[19] The original triviality results are contained in Lewis (1976) and Lewis (1986). For further investigation, see, for instance, Bradley (1999).

[20] For such an account, cp. McGee (1989).

[21] Strictly speaking, this will yield truth conditions only for non-iterated 'might'-statements. It would still be an open question whether such accounts extend to iterations of 'might' in a way consistent with Might.

[22] Cf. Barnett (2006) and Edgington (1995).

[23] I have borrowed the term 'partial assertion' from Price (1983) who develops a similar view concerning 'probably'.

9 Appendix: Proofs of Theorems

It needs to be shown that a sentence is always acceptable just in case it is a theorem of S5. The theorem can be split into two parts: soundness and completeness. I will only sketch how soundness is proven, since the proof is easy. The completeness proof, on the other hand, is presented in detail.

Proof of soundness (sketch). One needs to show the following statement:

> If P is an extended credence function, and A is a theorem of S5, then $P(A) = 1$.

Take, for instance, a standard axiomatization of S5 consisting of the standard axioms and the two rules modus ponens and necessitation. One first shows that the axioms are always assigned probability 1, and then that this property is preserved by the rules of inference.

As an example, take the axiom $A \supset \Diamond A$. Let P be any extended credence function. Now, either $P(A) = 0$ or $P(A) > 0$. In the former case, $P(\neg A) = 1$ and thus, by being a standard probability function, $P(A \supset \Diamond A) = 1$. In the latter case, $P(\Diamond A) = 1$ by Might. Hence, as above, $P(A \supset \Diamond A) = 1$. Similarly, one shows the other axioms to be always acceptable.

Also, let us focus on one of the rules of inference, necessitation, say. Let A be such that $P(A) = 1$ for all extended credence functions P. Take any such P. Then $P(\neg A) = 0$. By the satisfaction of Might, $P(\Diamond\neg A) = 0$. Hence, $P(\neg\Diamond\neg A) = 1$. Thus, we have shown that if A is always acceptable, then so is $\neg\Diamond\neg A$. Modus ponens is shown to be valid in a similar way. ■

Proof of completeness. It needs to be shown that there is, to any sentence which is not a theorem of S5, a probabilistic counterexample. More precisely,

> To any sentence A which is not a theorem of S5, there is an extended credence function P such that $P(A) < 1$.

Let us prove this by constructing the desired probability function out of a countermodel for A. So, assume that A is not a theorem of S5. Then there is a finite S5-adequate model $\mathcal{M} = \langle W, R, V\rangle$ which contains a world ω such that $\omega \not\models A$. Since $\mathcal{M}$ is S5-adequate, R is an equivalence relation. It can be assumed that every two worlds in W are accessible from each other.[24]

Now, let $p_1, \ldots, p_n$ be the atomic sentences occurring in A. To any world $x \in W$, define

$$S_x := \bigwedge_{x \models p_i} p_i \wedge \bigwedge_{x \not\models p_j} \neg p_j \,.$$

Given two worlds x and y, write $x \sim y$ if S_x and S_y are logically equivalent. With $\tilde{x}$ the equivalence class of x is denoted. The element $\overline{x}$ is a representative representing $\tilde{x}$, i.e. an arbitrary element in $\tilde{x}$. Moreover, let $\widetilde{W}$ be the set of representatives, i.e. it contains exactly one element out of each equivalence class.

[24] For the relevant facts of modal logic used here, see, for instance, Boolos (1993).

In order to define the intended probability function P, set

$$P(S_x) := \frac{|\tilde{x}|}{|W|}$$

and extend this to a probability function on the whole language in some way such that MIGHT is satisfied (this is always possible).[25] The point of our definition is that the family $(S_y)_{y\in\widetilde{W}}$ is a set of mutually exclusive sentences for which one has

(21) $$1 = P(\bigvee_{y\in\widetilde{W}} S_y) = \sum_{y\in\widetilde{W}} P(S_y).$$

Further, set to any world x and any sentence B

$$P_x := \frac{P(B \wedge S_x)}{P(S_x)}\,.$$

First of all, P_x is always well defined, since $P(S_x)$ is never zero. Think of P_x as resulting from P by conditionalizing on S_x. P_x may not satisfy MIGHT, but it is a standard probability function.

$P(A) < 1$ needs to be shown. This would finish the proof. In order to do so, let us prove the following statement:

(†) For all worlds $x \in W$ and all subsentences B of A, the following two conditions hold good:

  (i) If $x \models B$, then $P_x(B) = 1$, and
  (ii) If $x \not\models B$, then $P_x(B) = 0$.

This can be proven by induction on the complexity of B. Let x be any world.

$\underline{B = p_i}$: If $x \models p_i$, then

$$P_x(p_i) = \frac{P(p_i \wedge S_x)}{P(S_x)} = 1\,,$$

since, by definition, S_x implies p_i. Now, suppose $x \not\models p_i$. Since S_x implies $\neg p_i$, one has $P_x(p_i) = 0$.

$\underline{B = \neg C}$: If $x \models \neg C$, then $x \not\models C$. By induction hypothesis, we get $P_x(C) = 0$. This implies $P_x(\neg C) = 1$. The case $x \not\models \neg C$ is similar.

$\underline{B = C \wedge D}$: Assume $x \models C \wedge D$. Then $x \models C$ and $x \models D$. By induction hypothesis, this implies $P_x(C) = P_x(D) = 1$. It follows from this that $P_x(C \wedge D) = 1$. Now, suppose $x \not\models C \wedge D$. Then either $x \not\models C$ or $x \not\models D$. Assume $x \not\models C$. Then, by induction hypothesis, $P_x(C) = 0$. Hence, $P_x(C \wedge D) = 0$.

$\underline{B = \Diamond C}$: If $x \models \Diamond C$, then there is a world y such that $y \models C$. By induction hypothesis, one has $P_y(C) = 1$. This implies $P(C) > 0$. Since P satisfies MIGHT, $P(\Diamond C) = 1$. But this forces $P_x(\Diamond C) = 1$. Now, suppose $x \not\models \Diamond C$. Then, by our assumptions about the model $\mathcal{M}$, it is $y \not\models C$ for every $y \in W$ (it is at this stage where essential use is made of the fact that all worlds are

[25] By $|M|$, the cardinality of a set M is denoted.

accessible from each other). By induction hypothesis, for all $y \in W$: $P_y(C) = 0$. Hence, for all $y \in W$: $P(C \wedge S_y) = 0$. But, by (21),

$$P(C) = \sum_{y \in \widetilde{W}} P(C \wedge S_y) = 0\,.$$

Now, this implies $P(\Diamond C) = 0$. Therefore, $P_x(\Diamond C) = 0$ as well. This completes the proof of (†).

Finally, let us see how (†) allows finishing the proof. Since $\mathcal{M}$ is a countermodel of A, there is an $x \in W$ such that $x \not\models A$. By (†), $P_x(A) = 0$. This implies

(22) $$P(A \wedge S_x) = 0.$$

Now, using (21) and (22):

$$\begin{aligned} P(A) &= \sum_{y \in \widetilde{W}} P(A \wedge S_y) \\ &= \sum_{y \in \widetilde{W} \setminus \{\overline{x}\}} P(A \wedge S_y) \\ &\leq \sum_{y \in \widetilde{W} \setminus \{\overline{x}\}} P(S_y) \\ &= 1 - P(S_x) \\ &< 1\,. \end{aligned}$$

This finishes the proof. ■

Now to the proof of the deduction theorem. Recall that it goes like this:

> *An argument $\phi_1, \ldots, \phi_n \therefore \psi$ over a modal language is probabilistically valid just in case the sentence $(\phi_1 \wedge \ldots \phi_n) \supset \psi$ is always acceptable.*

For the sake of brevity, I will only sketch the proof.

Proof of the deduction theorem (sketch). The right-to-left direction can be proven directly from the standard axioms of probability; I will omit the argument. However, in proving the left-to-right direction one needs to take care of the fact that one is dealing with extended credence functions. So, by way of contraposition, let us assume that $(\phi_1 \wedge \ldots \wedge \phi_n) \supset \psi$ is not always acceptable. This means that there is an extended credence function P such that $P((\phi_1 \wedge \ldots \wedge \phi_n) \supset \psi) < 1$. Consequently,

(23) $P(\phi_1 \wedge \ldots \wedge \phi_n \wedge \neg\psi) > 0.$

A function * which maps a sentence from the modal language to a sentence of the non-modal part of it can be defined by the following recursive rules:

- For all propositional variables p_i: $p_i^* = p_i$.

- $(\neg\phi)^* = \neg\phi^*$.
- $(\phi \wedge \chi)^* = \phi^* \wedge \chi^*$.
- $(\Diamond\phi)^* = \neg\perp$ iff $P(\phi^*) > 0$ and $(\Diamond\phi)^* = \perp$ iff $P(\phi^*) = 0$.

Thus, the star leaves atomic sentences, negations, and conjunctions unchanged; it maps a modalized sentence on a tautology or a contradiction depending on whether P is positive on the sentence or not. By induction on the complexity of sentences, one shows $P(\phi \equiv \phi^*) = 1$. In particular, one derives

(24) $P(\phi) = P(\phi^*)$.

From (23) and (24) one infers

(25) $P(\phi_1^* \wedge \ldots \wedge \phi_n^* \wedge \neg\psi^*) > 0$.

Now, let $p_1, \ldots, p_m$ be the atomic sentences which occur in $\phi_1, \ldots, \phi_n, \psi$. Further, let SD be the set of state descriptions which can be formed from these atomic sentences, i.e. the sentences of the form $(\neg)p_1 \wedge (\neg)p_2 \wedge \ldots \wedge (\neg)p_m$. Let us say that a state description s *verifies* a non-modal sentence ϕ if ϕ is a logical consequence of s. By (25), there is a state description s in SD which verifies $\phi_1^* \wedge \ldots \wedge \phi_n^* \wedge \neg\psi^*$ such that

(26) $P(s) > 0$.

Let us choose a value x which equals 1 iff $P(s) = 1$ and which satisfies $1 > x > \frac{n}{n+1}$ otherwise. Note the following:

(27) $x > n(1 - x)$.

Let P' be any standard probability function over the non-modal part of the language which satisfies the following two constraints:

- $P'(s) = x$.
- $P'(r) > 0$ iff $P(r) > 0$ for all state descriptions r in SD.

Such a choice is always possible: start by defining P' over the state descriptions in SD; set $P(s) = x$ and distribute the remaining probability uniformly on the other state descriptions on which P is positive; finally, extend this function to the whole language.

Now, P' is a counterexample to the probabilistic validity of $\phi_1^*, \ldots, \phi_n^* \therefore \psi^*$, i.e.

(28) $u_{P'}(\psi^*) > \sum u_{P'}(\phi_i^*)$.

For, by the choice of x, the following chain of inequalities holds:

$$\sum u_{P'}(\phi_i^*) \leq n u_{P'}(\phi_1^* \wedge \ldots \wedge \phi_n^*) \leq n(1 - x) < x \leq P'(\neg\psi^*) = u_{P'}(\psi^*)\,.$$

Given this, extend P' to the modal part of the language in a way which satisfies Might. This results in an extended credence function P'' over the modal language which agrees with P' on the non-modal part of it. By induction on the complexity of sentences, one proves $P''(\phi \equiv \phi^*) = 1$ for all subsentences ϕ of the sentences $\phi_1, \ldots, \phi_n, \psi$ (here the second property of P' is used). In particular, one gets $P''(\phi) = P''(\phi^*) = P'(\phi^*)$, since ϕ^* is a non-modal sentence. By (28), P'' is now seen to be a counterexample to the probabilistic validity of our original inference $\phi_1, \ldots, \phi_n \therefore \psi$. ■

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